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On the basic properties of the negations generated by some parametric intuitionistic fuzzy implications

Piotr Dworniczak
Department of Applied Mathematics, University of Economics
al. Niepodległości 10, 60-967 Poznań, Poland
e-mail: *p.dworniczak@ue.poznan.pl*

Abstract: In this paper a class of intuitionistic parametric fuzzy implications and negations generated by them are presented. Fulfillment of some properties together with the Law of Excluded Middle and De Morgan's Law are investigated.

Keywords: Negation, Intuitionistic fuzzy implication, Law of Excluded Middle, De Morgan's Law.

1 Introduction

The Intuitionistic Fuzzy Logic (IFL) has been developed for about 25 years. Both theory and application of the IFL intrinsic to the Intuitionistic Fuzzy Sets (IFS) theory are being developed, starting with the publications of Krassimir Atanassov in the mid 1980s. In the IFL the truth-value of variable x is given by ordered pair $< a , b >$, where a, b, $a+b \in [0,1]$. The numbers a and b are interpreted as the degrees of validity and non-validity of the variable x. We denote the truth-value of x by $V(x)$.

The variable with truth-value *true* (in the classical logic) we denote by $\underline{1}$ and the variable *false* by $\underline{0}$. For this variables holds also $V(\underline{1}) = < 1 , 0 >$ and $V(\underline{0}) = < 0 , 1 >$.

We call the x an Intuitionistic Fuzzy Tautology (IFT), if and only if for $V(x) = < a , b >$ holds $a \geq b$ and, similar, an Intuitionistic Fuzzy co-Tautology (IFcT), if holds $a \leq b$.

For every x we can define the value of negation of x in the classical form $V(\neg_c x) = < b,a >$.

It is clear that an IFcT could be defined by IFT and $\neg_c$.

For $V(x) = < a , b >$ and $V(y) = < c , d >$ the conjunction $\wedge$ and disjunction $\vee$ we define by classical (following Atanassov [1]) formulas:

$$V(x \wedge y) = < \min\{a , c\} , \max\{b , d\} >,$$
$$V(x \vee y) = < \max\{a , c\} , \min\{b , d\} >,$$

and we denote relation $\preceq$ between the truth-values in the form: $V(x) \preceq V(y)$ if and only if $a \leq c$ and $b \geq d$.

An important operator of the IFL is an Intuitionistic Fuzzy Implication.

Definition 1.
The fuzzy implication (see [5, 6]) is a mapping $I : [0, 1]^2 \to [0, 1]$ where for $p_1, p_2, p, q_1, q_2, q \in [0, 1]$ holds:

(i1 FL) if $p_1 \le p_2$ then $I(p_1, q) \ge I(p_2, q)$,
(i2 FL) if $q_1 \le q_2$ then $I(p, q_1) \le I(p, q_2)$,
(i3 FL) $I(0, q) = 1$,
(i4 FL) $I(p, 1) = 1$,
(i5 FL) $I(1, 0) = 0$.

In the case of IFL the conditions (i1 FL)–(i5 FL) we would understand as:
(i1 IFL) if $V(x_1) \preceq V(x_2)$ then $V(x_1 \Rightarrow y) \succeq V(x_2 \Rightarrow y)$,
(i2 IFL) if $V(y_1) \preceq V(y_2)$ then $V(x \Rightarrow y_1) \preceq V(x \Rightarrow y_2)$,
(i3 IFL) $\underline{0} \Rightarrow y$ is an IFT,
(i4 IFL) $x \Rightarrow \underline{1}$ is an IFT,
(i5 IFL) $\underline{1} \Rightarrow \underline{0}$ is an IFcT.

In the literature (see [3]) more than a hundred different intuitionistic fuzzy implications are mentioned. L. Atanassova has presented an additional one, determined on the basis of an Atanassov's averaging operator @ (see [4]). This is a special case in a class of parametric intuitionistic fuzzy implications presented by Dworniczak [7].

The truth-value of the implication is given by formula

$$V(x \rightarrow_{\lambda} y) = \left< \frac{b+c+\lambda-1}{2\lambda}, \frac{a+d+\lambda-1}{2\lambda} \right>,$$

where $\lambda \in \Re, \lambda \ge 1$.

The second class of intuitionistic fuzzy parametric implications is given by formula

$$V(x \rightarrow_{\varphi} y) = \left< \frac{b+c+\varphi}{2\varphi}, \frac{a+d+\varphi-2}{2\varphi} \right>$$

where $\varphi \in \Re$, $\varphi \ge 2$ (a paper about the basic properties of these implication has been sent to an other journal and is awaiting for review).

The implications $\rightarrow_{\lambda}$ and $\rightarrow_{\varphi}$ fulfill Definition 1 with (i1 IFL)–(i5 IFL).

The above implications generate an intuitionistic fuzzy negations $\neg_{\lambda}$ and $\neg_{\varphi}$ respectively, with the truth-value

$$V(\neg_{\lambda} x) = \left< \frac{b+\lambda-1}{2\lambda}, \frac{a+\lambda}{2\lambda} \right>,$$

$$V(\neg_{\varphi} x) = \left< \frac{b+\varphi}{2\varphi}, \frac{a+\varphi-1}{2\varphi} \right>$$

The truth-values are determined on the basis of classical proposition $\neg x \Leftrightarrow x \rightarrow 0$ applied to the $\rightarrow_{\lambda}$ and $\rightarrow_{\varphi}$ implications.

Negations $\neg_{\lambda}$ and $\neg_{\varphi}$ are not involutive.

The special values of these implications and negations are:

$V(\underline{0} \rightarrow_{\lambda} \underline{0}) = \left< \frac{1}{2}, \frac{1}{2} \right>$, $V(\underline{1} \rightarrow_{\lambda} \underline{0}) = \left< \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda} \right>$, $V(\underline{0} \rightarrow_{\lambda} \underline{1}) = \left< \frac{\lambda+1}{2\lambda}, \frac{\lambda-1}{2\lambda} \right>$, $V(\underline{1} \rightarrow_{\lambda} \underline{1}) = \left< \frac{1}{2}, \frac{1}{2} \right>$ and $V(\underline{0} \rightarrow_{\varphi} \underline{0}) = \left< \frac{\varphi+1}{2\varphi}, \frac{\varphi-1}{2\varphi} \right>$, $V(\underline{1} \rightarrow_{\varphi} \underline{0}) = \left< \frac{1}{2}, \frac{1}{2} \right>$, $V(\underline{0} \rightarrow_{\varphi} \underline{1}) = \left< \frac{\varphi+2}{2\varphi}, \frac{\varphi-2}{2\varphi} \right>$, $V(\underline{1} \rightarrow_{\varphi} \underline{1}) = \left< \frac{\varphi+1}{2\varphi}, \frac{\varphi-1}{2\varphi} \right>$.

The special values of the negations are $V(\neg_\lambda \underline{1}) = < \frac{\lambda-1}{2\lambda}, \frac{\lambda+1}{2\lambda} >$, $V(\neg_\lambda \underline{0}) = < \frac{1}{2}, \frac{1}{2} >$ and $V(\neg_\varphi \underline{1}) = < \frac{1}{2}, \frac{1}{2} >$, $V(\neg_\varphi \underline{0}) = < \frac{\varphi+1}{2\varphi}, \frac{\varphi-1}{2\varphi} >$.

The values of the negations of the *fully ignorance* are $V(\neg_\lambda < 0, 0 >) = < \frac{\lambda-1}{2\lambda}, \frac{1}{2} >$ and $V(\neg_\varphi < 0, 0 >) = < \frac{1}{2}, \frac{\varphi-1}{2\varphi} >$. The first is an IFcT and the second an IFT.

Also, for $a \le 0{,}5$, $V(\neg_\lambda < a, a >) = < \frac{a+\lambda-1}{2\lambda}, \frac{a+\lambda}{2\lambda} >$ is an IFcT and $V(\neg_\varphi < a, a >) = < \frac{a+\varphi}{2\varphi}, \frac{a+\varphi-1}{2\varphi} >$ is an IFT.

It is easy to show that for every x $V(\neg_\lambda x)$ is an IFcT and $V(\neg_\varphi x)$ is an IFT.

The above implications and negations are therefore not a generalizations of the classical two-valued implications and negations.

2 Main results

In the literature (see [2, 4]) the fulfillment of the properties is considered

P1 $$x \Rightarrow \neg\neg x \tag{1}$$
P2 $$\neg\neg x \Rightarrow x \tag{2}$$
P3 $$\neg\neg\neg x = \neg x \tag{3}$$

Theorem 1

a) For implication $\rightarrow_\lambda$ and negation $\neg_\lambda$ none of the following properties is valid (in classical sense), also none of the formula (1)–(3) is an IFT.

b) For implication $\rightarrow_\varphi$ and negation $\neg_\varphi$ none of the following properties is valid (in classical sense), but the formulas (1)–(2) are the IFTs.

Proof. a) The truth-value of the $x \rightarrow_\lambda \neg_\lambda \neg_\lambda x$ is neither IFT nor IFcT.

Counterexample: let $\lambda = 1$ and $a = 1$, $b = 0$ then $x \rightarrow_\lambda \neg_\lambda \neg_\lambda x$ is IFcT instead for $a = 0$, $b = 1$ it is an IFT. It is similar simple to shown that $\neg_\lambda \neg_\lambda x \rightarrow_\lambda x$ is neither IFT nor IFcT and $\neg_\lambda \neg_\lambda \neg_\lambda x = \neg_\lambda x$ does not holds.

b)

Ad P1.

It is $V(x \rightarrow_\varphi \neg_\varphi \neg_\varphi x) = < \frac{4\varphi^2 b + a + 4\varphi^3 + 2\varphi^2 + \varphi - 1}{8\varphi^3}, \frac{4\varphi^2 a + b + 4\varphi^3 - 6\varphi^2 - \varphi}{8\varphi^3} >$.

We can easily check that for $a = b = 0{,}5$ and $\varphi = 2$ the above value is not equal to $< 1, 0 >$.

But $\frac{4\varphi^2 b + a + 4\varphi^3 + 2\varphi^2 + \varphi - 1}{8\varphi^3} \ge \frac{4\varphi^2 a + b + 4\varphi^3 - 6\varphi^2 - \varphi}{8\varphi^3}$ is equivalent to

$$(4\varphi^2-1)(b-a) \ge (2\varphi+1)(1-4\varphi)$$

and further to $b–a \geq \dfrac{1-4\varphi}{2\varphi-1}$ and these holds for $\varphi \geq 2$. Therefore $x \rightarrow_{\varphi} \neg_{\varphi} \neg_{\varphi} x$ is an IFT.

Ad P2.

It is $V(\neg_{\varphi} \neg_{\varphi} x \rightarrow_{\varphi} x) = < \dfrac{4\varphi^2 a + b + 4\varphi^3 + 2\varphi^2 - \varphi}{8\varphi^3}, \dfrac{4\varphi^2 b + a + 4\varphi^3 - 6\varphi^2 + \varphi - 1}{8\varphi^3} >$. We can easily check that for $a = b = 0{,}5$ and $\varphi = 2$ the above value is not equal to $< 1 , 0 >$. But

$$\frac{4\varphi^2 a + b + 4\varphi^3 + 2\varphi^2 - \varphi}{8\varphi^3} \geq \frac{4\varphi^2 b + a + 4\varphi^3 - 6\varphi^2 + \varphi - 1}{8\varphi^3}$$

is equivalent to $(4\varphi^2–1)(a–b) \geq -8\varphi^2 + 2\varphi - 1$ and further to $a–b \geq \dfrac{-8\varphi^2 + 2\varphi - 1}{4\varphi^2 - 1}$ and this holds because for $\varphi \geq 2$ there is (easy to check) $\dfrac{-8\varphi^2 + 2\varphi - 1}{4\varphi^2 - 1} \leq -1$.

Ad P3.

It is $V(\neg_{\varphi} \neg_{\varphi} \neg_{\varphi} x) = < \dfrac{b + 4\varphi^3 + 2\varphi^2 + \varphi}{8\varphi^3}, \dfrac{a + 4\varphi^3 - 2\varphi^2 + \varphi - 1}{8\varphi^3} >$.

It can be $\neg_{\varphi} \neg_{\varphi} \neg_{\varphi} x = \neg_{\varphi} x$ if $\dfrac{b + 4\varphi^3 + 2\varphi^2 + \varphi}{8\varphi^3} = \dfrac{b + \varphi}{2\varphi}$ and this never holds.

In the literature (see [2, 4]) the fulfillment by the intuitionistic fuzzy negations of the Law of Excluded Middle (LEM) in the basic form is considered

$$V(< a , b > \vee \neg < a , b >) = < 1 , 0 >, \tag{4}$$

or in the basic IFT-form (IF LEM)

$$V(< a , b > \vee \neg < a , b >) = < p , q >, \tag{5}$$

as well as in the Modified form (M LEM)

$$V(\neg \neg < a , b > \vee \neg < a , b >) = < 1 , 0 >, \tag{6}$$

or in the Modified IFT-form (M IF LEM)

$$V(\neg \neg < a , b > \vee \neg < a , b >) = < p , q >, \tag{7}$$

where $p \geq q$.

Since the negation is not necessarily (in general) involutive therefore LEM in the form (4) and (6) also in the form (5) and (7) need not be equivalent.

Theorem 2. The negation $\neg_{\lambda}$ does not satisfy any of the LEM form (4)–(7).

Proof by counterexample (for $\lambda = 1$) given by Atanassova in [4] is analogous also for $\lambda > 1$.

Theorem 3. The negation $\neg_{\varphi}$

a) does not satisfy the LEM (4) and M LEM (6)

b) satisfies IF LEM (5) and M IF LEM (7).

Proof: a) Counterexample: let $< a , b > = < 0{,}5 , 0{,}5 >$ and $\varphi = 2$. Then

$V(< a , b > \vee \neg_{\varphi} < a , b >) = V(\neg_{\varphi} \neg_{\varphi} < a , b > \vee \neg_{\varphi} < a , b >) = < 0{,}625 , 0{,}375 > \neq < 1 , 0 >$.

We note that $V(< a , b > \vee \neg_{\varphi} < a , b >) = < 1 , 0 >$ only for $< a , b > = < 1 , 0 >$ and $V(\neg_{\varphi} \neg_{\varphi} < a , b > \vee \neg_{\varphi} < a , b >)$ is never equal to $< 1 , 0 >$.

b)
IF LEM (5):

It is $V(<a, b> \vee \neg_{\varphi} <a, b>) = <\max\{a, \frac{b+\varphi}{2\varphi}\}, \min\{b, \frac{a+\varphi-1}{2\varphi}\}>$.

- If $a \in [0, 0.5]$ and $b \in [0, 0.5]$ then $\frac{b+\varphi}{2\varphi} \geq \frac{1}{2}$ therefore $\max\{a, \frac{b+\varphi}{2\varphi}\} = \frac{b+\varphi}{2\varphi}$ and

  $\min\{b, \frac{a+\varphi-1}{2\varphi}\} = \frac{1}{2}(b + \frac{a+\varphi-1}{2\varphi} - \left|b - \frac{a+\varphi-1}{2\varphi}\right|)$.

  Since $\frac{b+\varphi}{2\varphi} \geq \frac{1}{2}(b + \frac{a+\varphi-1}{2\varphi} - \left|b - \frac{a+\varphi-1}{2\varphi}\right|)$ is equivalent to

  $|\varphi(2b-1) - a + 1| \geq \varphi(2b-1) - 2b - (1-a)$ and this holds (the elements on the right side are non-positive), therefore $<a, b> \vee \neg_{\varphi} <a, b>$ is an IFT.

- If $a \in (0.5, 1]$ and $b \in [0, 0.5]$ then $\max\{a, \frac{b+\varphi}{2\varphi}\} > \frac{1}{2}$ while $\frac{a+\varphi-1}{2\varphi} = \frac{1}{2} + \frac{a-1}{2\varphi} \leq \frac{1}{2}$

  therefore $\min\{b, \frac{a+\varphi-1}{2\varphi}\} \leq \frac{1}{2}$, so $<a, b> \vee \neg_{\varphi} <a, b>$ is an IFT.

- If $a \in (0.5, 1]$ and $b \in (0.5, 1]$ then $\max\{a, \frac{b+\varphi}{2\varphi}\} = \frac{1}{2}(a + \frac{b+\varphi}{2\varphi} + \left|a - \frac{b+\varphi}{2\varphi}\right|)$

  and $\frac{a+\varphi-1}{2\varphi} \leq \frac{1}{2}$ therefore $\min\{b, \frac{a+\varphi-1}{2\varphi}\} = \frac{a+\varphi-1}{2\varphi}$.

  Since $\frac{1}{2}(a + \frac{b+\varphi}{2\varphi} + \left|a - \frac{b+\varphi}{2\varphi}\right|) \geq \frac{a+\varphi-1}{2\varphi}$ is equivalent to

  $2a(\varphi-1) + b + 1 + |\varphi(2a-1) - b| \geq \varphi-1$ and further to $(2a-1) + \frac{b+1}{\varphi-1} + \left|\frac{\varphi(2a-1)-b}{\varphi-1}\right| \geq 0$

  and this holds (the elements on the left side are non-negative),
  therefore $<a, b> \vee \neg_{\varphi} <a, b>$ is an IFT.

- If $a \in [0, 0.5]$ and $b \in (0.5, 1]$ then $\frac{b+\varphi}{2\varphi} > \frac{1}{2}$, therefore $\max\{a, \frac{b+\varphi}{2\varphi}\} = \frac{b+\varphi}{2\varphi}$ and

  $\frac{a+\varphi-1}{2\varphi} < \frac{1}{2}$, therefore $\min\{b, \frac{a+\varphi-1}{2\varphi}\} = \frac{a+\varphi-1}{2\varphi}$.

  Since $\frac{b+\varphi}{2\varphi} \geq \frac{a+\varphi-1}{2\varphi}$ holds, then $<a, b> \vee \neg_{\varphi} <a, b>$ is an IFT.

M IF LEM (7):

It is $V(\neg_{\varphi} \neg_{\varphi} <a, b> \vee \neg_{\varphi} <a, b>) = V(\neg_{\varphi} <a, b> \vee \neg_{\varphi} \neg_{\varphi} <a, b>) =$

$$= V(<\frac{b+\varphi}{2\varphi}, \frac{a+\varphi-1}{2\varphi}> \vee <\frac{\frac{a+\varphi-1}{2\varphi}+\varphi}{2\varphi}, \frac{\frac{b+\varphi}{2\varphi}+\varphi-1}{2\varphi}> =$$

$$= < \max\{\frac{b+\varphi}{2\varphi}, \frac{\frac{a+\varphi-1}{2\varphi}+\varphi}{2\varphi}\}, \min\{\frac{a+\varphi-1}{2\varphi}, \frac{\frac{b+\varphi}{2\varphi}+\varphi-1}{2\varphi}\} >.$$

Denoting $\frac{b+\varphi}{2\varphi} = w$ and $\frac{a+\varphi-1}{2\varphi} = z$, we obtain

$$V(\neg_\varphi \neg_\varphi < a, b > \vee \neg_\varphi < a, b >) = < \max\{w, \frac{z+\varphi}{2\varphi}\}, \min\{z, \frac{w+\varphi-1}{2\varphi}\} >$$

which is an IFT based on the proof given earlier for IF LEM.

Important in the classical, mathematical logic are also De Morgan's Laws given in the form

$$V(\neg x \wedge \neg y) = V(\neg(x \vee y)) \tag{8}$$

and

$$V(\neg x \vee \neg y) = V(\neg(x \wedge y)) \tag{9}$$

Two another forms of the De Morgan's Law are also mentioned, namely

$$V(\neg(\neg x \wedge \neg y)) = V(x \vee y) \tag{10}$$

$$V(\neg(\neg x \vee \neg y)) = V(x \wedge y) \tag{11}$$

For non-involutive negations the pairs (8):(10) and (9):(11) need not to be equivalent.

Theorem 4. The negations $\neg_\varphi$ and $\neg_\lambda$ satisfy the equalities (8) and (9) and do not satisfy the equalities (10) and (11).

Proof: In the case (8), for $\neg_\varphi$.

It is $V(\neg_\varphi x \wedge \neg_\varphi y) = V(< \frac{b+\varphi}{2\varphi}, \frac{a+\varphi-1}{2\varphi} > \wedge < \frac{d+\varphi}{2\varphi}, \frac{c+\varphi-1}{2\varphi} >) =$

$= < \min\{\frac{b+\varphi}{2\varphi}, \frac{d+\varphi}{2\varphi}\}, \max\{\frac{a+\varphi-1}{2\varphi}, \frac{c+\varphi-1}{2\varphi}\} > =$

$= < \frac{\min\{b,d\}+\varphi}{2\varphi}, \frac{\max\{a,c\}+\varphi-1}{2\varphi} > = V(\neg_\varphi (< \max\{a, c\}, \min\{b, d\} >)) =$

$= V(\neg_\varphi (x \vee y))$.

The proofs for another cases are analogous. The Theorem 4 in the case of λ=1 was first proved by Atanassova [4].

Multiple use of the negation $\neg_\lambda$ and $\neg_\varphi$ gives generally a lot of truth-values.

Let us denote $\neg_\lambda^1 x = \neg_\lambda x$ and $\neg_\lambda^{n+1} x = \neg_\lambda (\neg_\lambda^n x)$ (analogous for the $\neg_\varphi$).

Theorem 5. For a natural number $n \geq 1$ the negations $\neg_\lambda$ and $\neg_\varphi$ hold the relationships:

1) $V(\neg_\lambda^{2n-1} x) = < \frac{b}{(2\lambda)^{2n-1}} - \frac{\lambda+1}{(2\lambda+1)(2\lambda)^{2n-1}} + \frac{\lambda}{2\lambda+1}, \frac{a}{(2\lambda)^{2n-1}} - \frac{\lambda}{(2\lambda+1)(2\lambda)^{2n-1}} + \frac{\lambda+1}{2\lambda+1} >$,

2) $V(\neg_\lambda^{2n} x) = < \frac{a}{(2\lambda)^{2n}} - \frac{\lambda}{(2\lambda+1)(2\lambda)^{2n}} + \frac{\lambda}{2\lambda+1}, \frac{b}{(2\lambda)^{2n}} - \frac{\lambda+1}{(2\lambda+1)(2\lambda)^{2n}} + \frac{\lambda+1}{2\lambda+1} >$,

3) $V(\neg_{\varphi}^{2n-1} x) = < \frac{b}{(2\varphi)^{2n-1}} - \frac{\varphi}{(2\varphi+1)(2\varphi)^{2n-1}} + \frac{\varphi+1}{2\varphi+1}, \frac{a}{(2\varphi)^{2n-1}} - \frac{\varphi+1}{(2\varphi+1)(2\varphi)^{2n-1}} + \frac{\varphi}{2\varphi+1} >,$

4) $V(\neg_{\varphi}^{2n}(x)) = < \frac{a}{(2\varphi)^{2n}} - \frac{\varphi+1}{(2\varphi+1)(2\varphi)^{2n}} + \frac{\varphi+1}{2\varphi+1}, \frac{b}{(2\varphi)^{2n}} - \frac{\varphi}{(2\varphi+1)(2\varphi)^{2n}} + \frac{\varphi}{2\varphi+1} >.$

Proof. The proof is based on the principle of mathematical induction. The result for $\lambda = 1$ is given by Atanassova [4].

Corollary 1. $\lim_{n\to\infty} V(\neg_{\lambda}^{n} x) = < \frac{\lambda}{2\lambda+1}, \frac{\lambda+1}{2\lambda+1} >.$

Corollary 2. $\lim_{n\to\infty} V(\neg_{\varphi}^{n}(x)) = < \frac{\varphi+1}{2\varphi+1}, \frac{\varphi}{2\varphi+1} >.$

Corollary 3. $\lim_{\lambda\to\infty}(\lim_{n\to\infty} V(\neg_{\lambda}^{n} x)) = \lim_{\varphi\to\infty}(\lim_{n\to\infty} V(\neg_{\varphi}^{n}(x))) = < \frac{1}{2}, \frac{1}{2} >.$

Remark 1. $\lim_{n\to\infty} V(\neg_{\lambda}^{n} x)$ and $\lim_{n\to\infty} V(\neg_{\varphi}^{n}(x))$ are classical fuzzy sets.

Remark 2. $< \frac{1}{3}, \frac{2}{3} > \preceq \lim_{n\to\infty} V(\neg_{\lambda}^{n} x) \preceq < \frac{1}{2}, \frac{1}{2} > \preceq \lim_{n\to\infty} V(\neg_{\varphi}^{n} x) \preceq < \frac{3}{5}, \frac{2}{5} >.$

3 Conclusion

The negations, presented in this article, are, due to their simple formula, easy to apply. Only their basic properties are presented. Further research may involve comparing them with negations presented earlier in the literature (see references). The negation given above can be the basis for defining the complement of an intuitionistic fuzzy set and the relative complement, that is the set-theoretic difference of two IFSs ("subtraction" of the IFSs).

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