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THE HAUBER'S LAW IS AN INTUITIONISTIC FUZZY TAUTOLOGY

Krassimir Atanassov
Centre of Biomedical Engineering - Bulgarian Academy of Sciences, Acad. G. Bonchev str., Bl. 105, Sofia-1113, BULGARIA
e-mails: krat@bgcict.acad.bg krat@bgearn.acad.bg

For the needs of the below text, we will introduce some basic elements of the Intuitionistic Fuzzy Logic (IFL) following [1].

Thus two real numbers, $\mu(p)$ and $\nu(p)$, are assigned to each proposition p with the following constraint to hold:

$$\mu(p) + \nu(p) \leq 1.$$

They are called "truth degree" and "falsity degree" of the proposition.

Let this assignment be provided by an evaluation function V defined over a set of propositions S in such a way that:

$$V(p) = \langle \mu(p),\ \nu(p) \rangle.$$

Hence the function $V : S \to [0,1] \times [0,1]$ gives the truth and falsity degrees of all propositions in S.

We assume that the evaluation function V assigns to the logical truth T

$$V(T) = \langle 1, 0 \rangle,$$

and to the logical falsity F

$$V(F) = \langle 0, 1 \rangle.$$

The evaluation of the negation $\neg p$ of the proposition p is defined as:

$$V(\neg p) = \langle \nu(p), \mu(p) \rangle.$$

When the values $V(p)$ and $V(q)$ of the propositions p and q are known, the evaluation function V can be extended also for the operations "&", "$\vee$" and "$\supset$" through the definition :

$$V(p \& q) = \langle min(\mu(p), \mu(q)), max(\nu(p), \nu(q)) \rangle,$$

$$V(p \vee q) = \langle max(\mu(p), \mu(q)), min(\nu(p), \nu(q)) \rangle.$$

$$V(p \supset q) = \langle max(\nu(p), \mu(q)), min(\mu(p), \nu(q)) \rangle$$

Depending on the way of defining the "$\supset$" operation, in [1] two variants of IFPC are discussed.

It will be convenient to define for the propositions $p, q \in S$:

$$\neg V(p) = V(\neg p),$$

$$V(p) \wedge V(q) = V(p\&q),$$
$$V(p) \sqcup V(q) = V(p \vee q),$$
$$V(p) \rightarrow V(q) = V(p \supset q).$$

A propositional form A (cf. [2]: each proposition is a propositional form; if A is a propositional form then $\neg A$ is a propositional form; if A and B are propositional forms, then $A\&B$, $A \vee B$,$A \supset B$ are propositional forms) will be called a *tautology* if $V(A) = \langle 1, 0\rangle$, for all valuation functions V.

On the other hand, following [1], we will define the notion of *intuitionistic fuzzy tautology* (IFT) as follows:

$$\text{"}A \text{ is an IFT" } \underline{\text{ iff }} \ \mu(A) \geq \nu(A).$$

In [3] the following Hauber's law is formulated by

$$((p \supset q)\&(r \supset s)\&(p \sqcup r)\&\neg(q\&s)) \supset ((q \supset p)\&(s \supset r))$$

and it is proved that it is a standard tautology.

Here we shall prove the following

THEOREM: The Hauber's law is an IFT.

Proof: Let the propositions p, q, r, s be given and let everywhere:

$$V(p) =< a, b >,$$

$$V(q) =< c, d >,$$

$$V(r) =< e, f >,$$

$$V(s) =< g, h > .$$

Then

$$\begin{aligned}
&V(((p \supset q)\&(r \supset s)\&(p \sqcup r)\& \quad \neg(q\&s)) \supset ((q \supset p)\&(s \supset r)))\\
&= ((< a, b > \rightarrow < c, d >) \wedge (< e, f > \rightarrow < g, h >) \wedge (< a, b > \vee < e, f >)\wedge\\
&\quad \neg(< c, d > \wedge < g, h >))\rightarrow((< c, d > \rightarrow < a, b >) \wedge (< g, h > \rightarrow < e, f >))\\
&= (< max(b, c), min(a, d) > \wedge < max(f, g), min(e, h) > \wedge < max(a, e), min(b, f) >\\
&\quad \wedge < max(d, h), min(c, g) >)\rightarrow(< max(a, d), min(b, c) > \wedge < max(e, h), min(f, g) >)\\
&= (< min(max(b, c), max(f, g), max(a, e), max(d, h)), max(min(a, d), min(e, h), min(b, f),\\
&\quad min(c, g)) >)\rightarrow < min(max(a, d), max(e, h)), max(min(b, c), min(f, g)) >\\
&=< max(min(a, d), min(e, h), min(b, f), min(c, g), min(max(a, d), max(e, h))),\\
&\quad min(max(b, c), max(f, g), max(a, e), max(d, h), max(min(b, c), min(f, g))) >
\end{aligned}$$

Let

$$\begin{aligned}
A &\equiv max(min(a, d), min(e, h), min(b, f), min(c, g), min(max(a, d), max(e, h)))\\
&\quad - min(max(b, c), max(f, g), max(a, e), max(d, h), max(min(b, c), min(f, g)))\\
&\geq max(min(a, d), min(e, h), min(max(a, d), max(e, h))) - min(max(a, e), max(d, h)),
\end{aligned}$$

i.e.,

$$A \geq max(min(a, d), min(e, h), min(max(a, d), max(e, h))) - min(max(a, e), max(d, h)).$$

If $a \geq d \geq e \geq h$, then: $A \geq max(d, h, min(a, e)) - min(a, d) = d - d = 0.$

If $a \geq d \geq h \geq e$, then: $A \geq max(d, e, min(a, h)) - min(a, d) = d - d = 0.$

If $a \geq e \geq d \geq h$, then: $A \geq max(d, h, min(a, e)) - min(a, d) = d - d = 0.$

If $a \geq e \geq h \geq d$, then: $A \geq max(d, h, min(a, e)) - min(a, h) = e - h \geq 0$.
If $a \geq h \geq d \geq e$, then: $A \geq max(d, e, min(a, h)) - min(a, h) = h - h = 0$.
If $a \geq h \geq e \geq d$, then: $A \geq max(d, e, min(a, h)) - min(a, h) = h - h = 0$.
If $d \geq a \geq e \geq h$, then: $A \geq max(a, h, min(d, e)) - min(a, d) = a - a = 0$.
If $d \geq a \geq h \geq e$, then: $A \geq max(a, e, min(d, h)) - min(a, d) = a - a = 0$.
If $d \geq e \geq a \geq h$, then: $A \geq max(a, h, min(d, e)) - min(e, d) = e - e = 0$.
If $d \geq e \geq h \geq a$, then: $A \geq max(a, h, min(d, e)) - min(e, d) = e - e = 0$.
If $d \geq h \geq a \geq e$, then: $A \geq max(a, e, min(d, h)) - min(a, d) = h - a \geq 0$.
If $d \geq h \geq e \geq a$, then: $A \geq max(a, e, min(d, h)) - min(e, d) = h - e \geq 0$.
If $e \geq a \geq d \geq h$, then: $A \geq max(d, h, min(a, e)) - min(e, d) = a - d \geq 0$.
If $e \geq a \geq h \geq d$, then: $A \geq max(d, h, min(a, e)) - min(e, h) = a - h \geq 0$.
If $e \geq d \geq a \geq h$, then: $A \geq max(a, h, min(d, e)) - min(e, d) = d - d = 0$.
If $e \geq d \geq h \geq a$, then: $A \geq max(a, h, min(d, e)) - min(e, d) = d - d = 0$.
If $e \geq h \geq a \geq d$, then: $A \geq max(d, h, min(a, e)) - min(e, h) = h - h = 0$.
If $e \geq h \geq d \geq a$, then: $A \geq max(a, h, min(d, e)) - min(e, h) = h - h = 0$.
If $h \geq a \geq e \geq d$, then: $A \geq max(d, e, min(a, h)) - min(a, h) = a - a = 0$.
If $h \geq a \geq d \geq e$, then: $A \geq max(d, e, min(a, h)) - min(a, h) = a - a = 0$.
If $h \geq d \geq a \geq e$, then: $A \geq max(a, e, min(d, h)) - min(a, h) = d - a \geq 0$.
If $h \geq d \geq e \geq a$, then: $A \geq max(a, e, min(d, h)) - min(e, h) = d - e \geq 0$.
If $h \geq e \geq a \geq d$, then: $A \geq max(d, e, min(a, h)) - min(e, h) = e - e = 0$.
If $h \geq e \geq d \geq a$, then: $A \geq max(a, e, min(d, h)) - min(e, h) = e - e = 0$.
Therefore, the Hauber's lay is an IFT.

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